*Int J Software Informatics,* Volume 8, Issue 3-4 (2014), pp. 277–288
International Journal of Software and Informatics, ISSN 1673-7288

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Tel: +86-10-62661040

# Decorrelating Capabilities and Measurement-Induced Correlations of Unruh Effect in Dirac Field

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**Abstract** We revisit the issue of information transfer induced by Unruh effect in a free Dirac field, where one of the observers (Rob) is uniformly accelerated with respect to his partner (Alice), each holding a mode of a free Dirac field in Minkowski spacetime. We first introduce the information loss induced by Unruh effect when the initial states between the two modes shared by Alice and Rob are any Bell-diagonal states, and give their analytic expressions. Then from the decorrelating capabilities and measurement-induced correlations perspectives, we reinterpret the changes of correlations induced by Unruh effect in Dirac field, which may shed new light on the understanding of Unruh effect from the information theory viewpoint.
**Key words:** Unruh effect; decorrelating capabilities; measurement-induced correlations; quantum discord; quantum mutual information; entanglement of formation

**Li N. Decorrelating capabilities and measurement-induced correlations of unruh effect in dirac field.** *Int J Software Informatics*, **Vol.8, No.3-4 (2014): 277–288. http://www.ijsi.org/1673-7288/8/i196.htm**

## 1 Introduction

The relativistic quantum information theory, which combines the theory of relativity, quantum theory, and information theory, leads us to a deeper interpretation of physical world[4,21]. One of the most important examples is that it supplies a new way to understand the information paradox when black holes are involved[28,29].

How to understand Unruh effect from the information theory viewpoint is another fundamental question in the relativistic quantum information theory. Diverse efforts have been made to investigate the dynamics of teleportation fidelity[2], quantum entanglement[1,10], quantum discord[7,24], Bell nonlocality[9], and quantum Fisher information[27] under Unruh effect in Dirac fields or scale fields.

This work is sponsored by the National Natural Science Foundation of China (Grants No. 11375259, No. 11405262 and No. 61134008), the National Center for Mathematics and Interdisciplinary Sciences, Chinese Academy of Sciences (Grant No. Y029152K51), and Key Laboratory of Random Complex Structures and Data Science, CAS (Grant No. 2008DP173182).
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Received 2014-10-31; Accepted 2014-12-28.

Most of the above results start with the initial states between the two modes shared by the inertial observer and the accelerated observer in Minkowski spacetime to be maximally entangled states. We will generalize the results by starting with any Bell-diagonal states and investigate the information loss induced by Unruh effect in a free Dirac field.

To illustrate the Unruh effect, suppose one observer, Alice, stays in an inertial frame while her partner Rob undergoes uniform acceleration $a$, each holding a mode of a free Dirac field in Minkowski spacetime. In order to describe what Rob perceives from his perspective, we should transform from the Minkowski coordinates $(t, z)$ to the Rindler coordinates $(\tau, \xi)$:

$$\begin{aligned} &at = e^{a\xi}\sinh(a\tau), \quad az = e^{a\xi}\cosh(a\tau), \quad |z| < t, && \text{I} \\ &at = -e^{a\xi}\sinh(a\tau), \quad az = -e^{a\xi}\cosh(a\tau), \quad |z| > t, && \text{II} \end{aligned}$$

which defines the right (region I) and left (region II) Rindler wedges. Usually we refer to the accelerating observers in regions I and II as Rob and anti-Rob, respectively. According to the Bogoliubov transformation, the Minkowski vacuum state and single excitation state can be expressed in terms of Rindler modes

$$|0_k\rangle \to \cos r|0_k\rangle^I|0_{-k}\rangle^{II} + \sin r|1_k\rangle^I|1_{-k}\rangle^{II}, \tag{1}$$

$$|1_k\rangle \to |1_k\rangle^I|0_{-k}\rangle^{II}. \tag{2}$$

Here $|n_k\rangle^I$ and $|n_{-k}\rangle^{II}$ $(n = 0, 1)$ refer to the modes corresponding to the Rindler region I and II, respectively. $\cos r = (e^{-2\pi\omega c/a} + 1)^{-1/2}$ with $\omega = |k|$. The parameter $r \in [0, \pi/4)$ is a monotonically increasing function of the acceleration $a \in [0, \infty)$, and $r \to \pi/4$, as $a \to \infty$. For simplicity, we refer to the particle mode $|n_k\rangle^I$ as $|n\rangle^I$ and the antiparticle mode $|n_{-k}\rangle^{II}$ as $|n\rangle^{II}$ .

Note that a particle undergoing eternal uniform acceleration remains constrained to either Rindler region I or II and has no access to the opposite region, since these two regions are casually disconnected. When we consider the modes in region I, we have to trace off the modes in the region II, which implies that Unruh effect could be considered as a quantum channel from the system R to the system I, denoted by $\mathcal{E}$. On the other hand, if we trace off the modes in the region I, we get the complementary Unruh channel $\mathcal{E}_c$ from the system R to the system II.

This paper is structured as follows. In Sec. 2, we review and specify the measures of classical and quantum correlations, which will be our main tools in quantifying the information loss, the decorrelating capabilities and measurement-induced correlations for Unruh effect. In Sec. 3, we first introduce the information loss induced by Unruh effect when the initial states between Alice and Rob's modes are any Bell-diagonal states, and then calculate the decorrelating capabilities for Unruh Channel and complementary Unruh channel. In Sec. 4, we calculate the measurement-induced correlations of Unruh effect in Dirac field. Finally, we conclude in Sec. 5.

## 2 Classical and Quantum Correlations in Bipartite States

Given a bipartite state $\rho^{AB}$ shared by two parties $A$ and $B$ with marginal states $\rho^A := \mathrm{tr}_B\rho^{AB}$ and $\rho^B := \mathrm{tr}_A\rho^{AB}$, its amount of total correlations is usually quantified by the quantum mutual information[12, 15, 22, 23]

$$I(\rho^{AB}) := S(\rho^A) + S(\rho^B) - S(\rho^{AB}).$$

Here $S(\rho^A) := -\mathrm{tr}\rho^A \log_2 \rho^A$ is the von Neumann entropy.

Following Henderson and Vedral[13], the amount of classical correlations in $\rho^{AB}$ is well quantified by

$$C(\rho^{AB}) := \max_{\Pi} \left[ S(\rho^A) - \sum_j q_j S(\rho_j^A) \right],$$

where the max is over all measurements $\Pi = \{\Pi_j\}$ on the system $B$, and $q_j := \mathrm{tr}(\mathbf{1} \otimes \Pi_j)\rho^{AB}(\mathbf{1} \otimes \Pi_j^\dagger)$, $\rho_j^A := \mathrm{tr}_B(\mathbf{1} \otimes \Pi_j)\rho^{AB}(\mathbf{1} \otimes \Pi_j^\dagger)/q_j$. Since $I(\rho^{AB})$ quantifies the total correlations, the amount of quantum correlations can be defined as

$$Q(\rho^{AB}) := I(\rho^{AB}) - C(\rho^{AB}).$$

In particular, if the measurements are restricted to the von Neumann measurements (orthogonal, one dimensional projections) in the above definition, then one gets the quantum discord introduced by Ollivier and Zurek[20], which has operational interpretations and interesting applications in quantum information theory[8,17,25]. Except for some particular states such as the Bell-diagonal states[19], it is usually difficult to evaluate the classical correlations and the quantum discord, even for two-qubit states[6,11].

Another important and by far the best studied measure of a particular kind of quantum correlations (entanglement) is the entanglement of formation[3]: $E(\rho^{AB}) := \min \sum_k r_k E(|\Psi_k^{AB}\rangle\langle\Psi_k^{AB}|)$. Here the min is over all pure state decompositions $\rho^{AB} = \sum_k r_k |\Psi_k^{AB}\rangle\langle\Psi_k^{AB}|$, and $E(|\Psi_k^{AB}\rangle\langle\Psi_k^{AB}|) = S(\mathrm{tr}_B(|\Psi_k^{AB}\rangle \langle\Psi_k^{AB}|)$ is the entanglement entropy of the pure state $|\Psi_k^{AB}\rangle$. In particular, for any two-qubit state $\rho^{AB}$, its entanglement of formation can be explicitly evaluated as[26]

$$E(\rho^{AB}) = H\left( \left\{ \frac{1-\sqrt{1-\lambda^2}}{2}, \frac{1+\sqrt{1-\lambda^2}}{2} \right\} \right).$$

Here $H(\{p_i\}) := -\sum p_i \log_2 p_i$ is the Shannon entropy function of the probability distribution $\{p_i\}$, $\lambda := \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$ is the concurrence of $\rho^{AB}$, $\lambda_j$ are the eigenvalues of $\rho^{AB}\tilde{\rho}^{AB}$ in decreasing order, $\tilde{\rho}^{AB} := (\sigma_y \otimes \sigma_y)\bar{\rho}^{AB}(\sigma_y \otimes \sigma_y)$ is the time-reversed version of $\rho^{AB}$, while $\bar{\rho}^{AB}$ is the complex conjugate of $\rho^{AB}$ (in matrix form), $\sigma_y$ is the Pauli spin matrix.

The elegant Koashi-Winter relation[14]

$$S(\rho^B) = C(\rho^{BC}) + E(\rho^{AB}) \tag{3}$$

connects the classical correlations in the $BC$ system and the entanglement of formation in the $AB$ system for any pure tripartite state $\rho^{ABC}$.

## 3 Decorrelating Capabilities of Unruh Effect

In this section, we first generalize the initial state between Alice and Rob from maximally entangled state to any Bell-diagonal states. Assume Alice and Rob share a Bell-diagonal initial state

$$\rho^{AR} = \frac{1}{4}\left(\mathbf{1}^{AR} + \sum_{j=1}^{3} c_j \sigma_j^A \otimes \sigma_j^R\right),$$

where $\mathbf{1}^{AR}$ is the identity operator in the Hilbert space of the two qubits $A$ and $R$, $\sigma_j^A$ and $\sigma_j^R$ are the Pauli operators of the qubits $A$ and $R$, and $c = (c_1, c_2, c_3)$ is a real vector satisfying the unit trace and positivity conditions of the density operators $\rho^{AR}$. In the standard basis, $\rho^{AR}$ can be expressed as

$$\rho^{AR} = \frac{1}{4}\begin{pmatrix} c_3^+ & 0 & 0 & c^- \\ 0 & c_3^- & c^+ & 0 \\ 0 & c^+ & c_3^- & 0 \\ c^- & 0 & 0 & c_3^+ \end{pmatrix}$$

where $c_3^+ = 1 + c_3$, $c_3^- = 1 - c_3$, $c^+ = c_1 + c_2$, and $c^- = c_1 - c_2$. It can be directly checked that $\rho^{\mathrm{AR}}$ has eigenvalues

$$\lambda_1^{AR} = \frac{c_3^+ + c^-}{4}, \quad \lambda_2^{AR} = \frac{c_3^+ - c^-}{4}, \quad \lambda_3^{AR} = \frac{c_3^- + c^+}{4}, \quad \lambda_4^{AR} = \frac{c_3^- - c^+}{4},$$

from which we readily see the constraints of the coefficients $c_j$ are such that $\lambda_j^{AR} \in [0, 1]$, for $j = 1, 2, 3, 4$. The marginal states of $\rho^{AR}$ are $\rho^A = \mathbf{1}/2$ and $\rho^R = \mathbf{1}/2$.

When Alice stays stationary and Rob moves with uniform acceleration $a$, using Eqs. (1) and (2), we can rewrite the state $\rho^{AR}$ in terms of Minkowski modes for Alice and Rindler modes for Rob. Since Rob is causally disconnected from the region II, the only information which is physically accessible to the observers is encoded in the Minkowski modes A described by Alice and the Rindler modes I described by Rob. Let $\mathcal{I}$ be the identity operation on the system $A$. Taking the trace over the modes in region II, we obtain

$$\begin{aligned} \rho^{A,I} &= \mathcal{I} \otimes \mathcal{E}(\rho^{AR}) \\ &= \frac{1}{4}\begin{pmatrix} c_3^+ \cos^2 r & 0 & 0 & c^- \cos r \\ 0 & c_3^+ \sin^2 r + c_3^- & c^+ \cos r & 0 \\ 0 & c^+ \cos r & c_3^- \cos^2 r & 0 \\ c^- \cos r & 0 & 0 & c_3^- \sin^2 r + c_3^+ \end{pmatrix}, \end{aligned}$$

in the standard basis, with its eigenvalues

$$\lambda_{1,2}^{A,I} = \frac{1 - c_3 \cos^2 r \pm \sqrt{\sin^4 r + (c_1 + c_2)^2 \cos^2 r}}{4},$$
$$\lambda_{3,4}^{A,I} = \frac{1 + c_3 \cos^2 r \pm \sqrt{\sin^4 r + (c_1 - c_2)^2 \cos^2 r}}{4}.$$

Similarly, taking the trace over the modes in region I, we obtain

$$\begin{aligned}\rho^{A,II} &= \mathcal{I} \otimes \mathcal{E}_c(\rho^{AR}) \\ &= \frac{1}{4}\begin{pmatrix} c_3^+ \cos^2 r + c_3^- & 0 & 0 & c^+ \sin r \\ 0 & c_3^+ \sin^2 r & c^- \sin r & 0 \\ 0 & c^- \sin r & c_3^- \cos^2 r + c_3^+ & 0 \\ c^+ \sin r & 0 & 0 & c_3^- \sin^2 r \end{pmatrix},\end{aligned}$$

in the standard basis, with its eigenvalues

$$\begin{aligned}\lambda_{1,2}^{A,II} &= \frac{1 - c_3 \sin^2 r \pm \sqrt{\cos^4 r + (c_1 + c_2)^2 \cos^2 r}}{4}, \\ \lambda_{3,4}^{A,II} &= \frac{1 + c_3 \sin^2 r \pm \sqrt{\cos^4 r + (c_1 - c_2)^2 \cos^2 r}}{4},\end{aligned}$$

and the marginal states are

$$\rho^A = \frac{\mathbf{1}}{2}, \quad \rho^I = \mathcal{E}(\rho^R) = \frac{1}{2}\begin{pmatrix} \cos^2 r & 0 \\ 0 & 1 + \sin^2 r \end{pmatrix},$$

$$\rho^{II} = \mathcal{E}_c(\rho^R) = \frac{1}{2}\begin{pmatrix} 1 + \cos^2 r & 0 \\ 0 & \sin^2 r \end{pmatrix}.$$

Consequently, the quantum mutual information in $\rho^{AR}$, $\rho^{A,I}$, and $\rho^{A,II}$ can be calculated as

$$\begin{aligned}I(\rho^{AR}) &= S(\rho^A) + S(\rho^R) - S(\rho^{AR}) \\ &= 2 - H(\{\lambda_j^{AR}\}), \\ I(\rho^{A,I}) &= S(\rho^A) + S(\rho^I) - S(\rho^{A,I}) \\ &= 1 + H(\{\lambda^I, 1 - \lambda^I\}) - H(\{\lambda_j^{A,I}\}), \\ I(\rho^{A,II}) &= S(\rho^A) + S(\rho^{II}) - S(\rho^{A,II}) \\ &= 1 + H(\{\lambda^{II}, 1 - \lambda^{II}\}) - H(\{\lambda_j^{A,II}\}),\end{aligned}$$

respectively, with $\lambda^I = \frac{\cos^2 r}{2}$, $\lambda^{II} = \frac{\sin^2 r}{2}$, and $H(\cdot)$ denotes the Shannon entropy of the probability distribution.

By the monotonicity of quantum mutual information under operations, we know

$$I(\rho^{A,I}) \leqslant I(\rho^{AR}), \quad I(\rho^{A,II}) \leqslant I(\rho^{AR}).$$

Therefore, we could define the information loss induced by Unruh effect in Dirac field as

$$\begin{aligned}L^I(r, c_1, c_2, c_3) &:= I(\rho^{AR}) - I(\rho^{A,I}) \\ &= (S(\rho^A) + S(\rho^R) - S(\rho^{AR})) - (S(\rho^A) + S(\rho^I) - S(\rho^{A,I})) \\ &= S(\rho^R) + S(\rho^{A,I}) - S(\rho^{AR}) - S(\rho^I) \\ &= 1 + H(\{\lambda_j^{A,I}\}) - H(\{\lambda^I, 1 - \lambda^I\}) - H(\{\lambda_j^{AR}\}).\end{aligned}$$

We plot $L^I(r, c_1, c_2, c_3)$ via $r$, when $c = (1, -1, 1), (0, 0, 1), (1, 0, 0)$, and $(1/3, 1/3, 1/3)$ in Fig. 1.

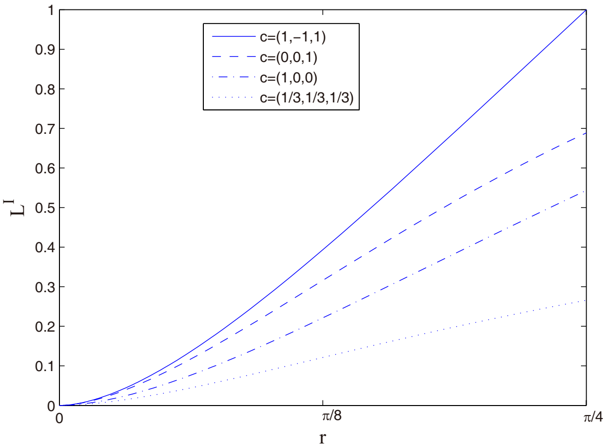


Figure 1. The information loss induced by Unruh effect.

On the other hand, we define the information loss induced by the complementary Unruh channel as

$$
\begin{aligned}
L^{II}(r,c_1,c_2,c_3) :&= I(\rho^{AR}) - I(\rho^{A,II}) \\
&= (S(\rho^A) + S(\rho^R) - S(\rho^{\mathrm{AR}})) - (S(\rho^A) + S(\rho^{II}) - S(\rho^{A,II})) \\
&= S(\rho^R) + S(\rho^{A,II}) - S(\rho^{AR}) - S(\rho^{II}) \\
&= 1 + H(\{\lambda_j^{A,II}\}) - H(\{\lambda^{II}, 1-\lambda^{II}\}) - H(\{\lambda_j^{AR}\}).
\end{aligned}
$$

We plot $L^{II}(r,c_1,c_2,c_3)$ via $r$, when $c = (1,-1,1), (0,0,1), (1,0,0)$, and $(1/3,1/3,1/3)$ in Fig. 2.

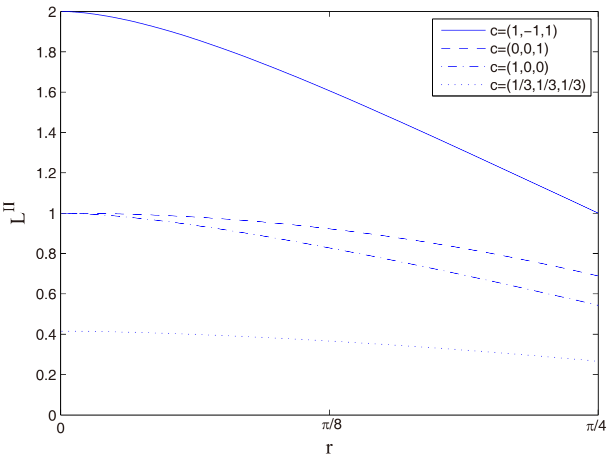


Figure 2. The information loss induced by the complementary Unruh channel.

Now recall the definition of decorrelating capabilities for a channel $\mathcal{E}$ performed on the state $\rho^B$ in system B[5,16]. Let $\rho^{AB} = |\Psi^{AB}\rangle\langle\Psi^{AB}|$ be a purification of $\rho^B$, and $\mathcal{I}$ be the identity operation on the system $A$. Put $\rho^{A'B'} = \mathcal{I} \otimes \mathcal{E}(\rho^{AB})$, then the total decorrelating capability of the channel $\mathcal{E}$ is defined by

$$D(\rho^B,\mathcal{E}) = I(\rho^{AB}) - I(\rho^{A'B'}),$$

which is the loss of total (classical + quantum) correlations in the purification $\rho^{AB}$ of $\rho^B$ induced by the operation $\mathcal{E}$. The classical decorrelation of $\mathcal{E}$ with respect to $\rho^B$ is defined as

$$D_c(\rho^B,\mathcal{E}) = C(\rho^{AB}) - C(\rho^{A'B'}),$$

and the quantum decorrelation of $\mathcal{E}$ with respect to $\rho^B$ is defined as

$$D_q(\rho^B,\mathcal{E}) = Q(\rho^{AB}) - Q(\rho^{A'B'}).$$

The intuitive meaning of these decorrelation measures is clear: $D_c(\rho^B,\mathcal{E})$ quantifies the loss of classical correlations induced by $\mathcal{E}$ , while $D_q(\rho^B,\mathcal{E})$ quantifies the loss of quantum correlations. By the definitions, we apparently have

$$D(\rho^B,\mathcal{E}) = D_c(\rho^B,\mathcal{E}) + D_q(\rho^B,\mathcal{E}).$$

To get rid of their dependence on $\rho^B$, Luo *et. al*[16] used the maximally mixed state to define

$$D(\mathcal{E}) = D(\frac{\mathbf{1}}{d^B},\mathcal{E}), \quad D_c(\mathcal{E}) = D_c(\frac{\mathbf{1}}{d^B},\mathcal{E}), \quad D_q(\mathcal{E}) = D_q(\frac{\mathbf{1}}{d^B},\mathcal{E}),$$

as measures of the total decorrelating capability, classical decorrelating capability, and quantum decorrelating capability of $\mathcal{E}$, respectively. Here $d^B$ denotes the dimension of the system B.

When $c_1 = 1$, $c_2 = -1$, $c_3 = 1$, the initial state $\rho^{AR}$ is restricted to maximally entangled state, which is pure and its marginal state is the maximally mixed state $\rho^R = \frac{\mathbf{I}}{2}$. The entanglement of formations for the states $\rho^{A,I}$, $\rho^{A,II}$, and $\rho^{I,II}$ have been calculated in the Ref. [1],

$$\begin{aligned}
E(\rho^{A,I}) =& -\frac{1+\sin r}{2}\log_2\frac{1+\sin r}{2} - \frac{1-\sin r}{2}\log_2\frac{1-\sin r}{2},\\
E(\rho^{A,II}) =& -\frac{1+\cos r}{2}\log_2\frac{1+\cos r}{2} - \frac{1-\cos r}{2}\log_2\frac{1-\cos r}{2},\\
E(\rho^{I,II}) =& -\frac{1+\sqrt{1-\sin^2 r\cos^2 r}}{2}\log_2\frac{1+\sqrt{1-\sin^2 r\cos^2 r}}{2}\\
& -\frac{1-\sqrt{1-\sin^2 r\cos^2 r}}{2}\log_2\frac{1-\sqrt{1-\sin^2 r\cos^2 r}}{2}.
\end{aligned}$$

By Koashi-Winter equality (3)[14], we get the expressions for the total, classical, and quantum correlations for the states $\rho^{A,I}$, $\rho^{A,II}$

$$\begin{aligned}
I(\rho^{A,I}) =& S(\rho^A) + S(\rho^I) - S(\rho^{A,I})\\
=& 1 - \frac{\cos^2 r}{2}\log_2\frac{\cos^2 r}{2} - \frac{1+\sin^2 r}{2}\log_2\frac{1+\sin^2 r}{2}\\
& + \frac{\sin^2 r}{2}\log_2\frac{\sin^2 r}{2} + \frac{1+\cos^2 r}{2}\log_2\frac{1+\cos^2 r}{2},\\
I(\rho^{A,II}) =& - S(\rho^A) + S(\rho^{II}) - S(\rho^{A,II})
\end{aligned}$$

$$
\begin{aligned}
&=1-\frac{\sin^2 r}{2}\log_2\frac{\sin^2 r}{2}-\frac{1+\cos^2 r}{2}\log_2\frac{1+\cos^2 r}{2}\\
&\quad+\frac{\cos^2 r}{2}\log_2\frac{\cos^2 r}{2}+\frac{1+\sin^2 r}{2}\log_2\frac{1+\sin^2 r}{2},\\
C(\rho^{A,I}) &=S(\rho^A)-E(\rho^{A,II})\\
&=1+\frac{1+\cos r}{2}\log_2\frac{1+\cos r}{2}+\frac{1-\cos r}{2}\log_2\frac{1-\cos r}{2},\\
C(\rho^{A,II}) &=S(\rho^A)-E(\rho^{A,I})\\
&=1+\frac{1+\sin r}{2}\log_2\frac{1+\sin r}{2}+\frac{1-\sin r}{2}\log_2\frac{1-\sin r}{2},\\
Q(\rho^{A,I}) &=S(\rho^I)+E(\rho^{A,II})-S(\rho^{A,I})\\
&=-\frac{\cos^2 r}{2}\log_2\frac{\cos^2 r}{2}-\frac{1+\sin^2 r}{2}\log_2\frac{1+\sin^2 r}{2}\\
&\quad-\frac{1+\cos r}{2}\log_2\frac{1+\cos r}{2}-\frac{1-\cos r}{2}\log_2\frac{1-\cos r}{2}\\
&\quad+\frac{\sin^2 r}{2}\log_2\frac{\sin^2 r}{2}+\frac{1+\cos^2 r}{2}\log_2\frac{1+\cos^2 r}{2},\\
Q(\rho^{A,II}) &=S(\rho^{II})+E(\rho^{A,I})-S(\rho^{A,II})\\
&=-\frac{\sin^2 r}{2}\log_2\frac{\sin^2 r}{2}-\frac{1+\cos^2 r}{2}\log_2\frac{1+\cos^2 r}{2}\\
&\quad-\frac{1+\sin r}{2}\log_2\frac{1+\sin r}{2}-\frac{1-\sin r}{2}\log_2\frac{1-\sin r}{2}\\
&\quad+\frac{\cos^2 r}{2}\log_2\frac{\cos^2 r}{2}+\frac{1+\sin^2 r}{2}\log_2\frac{1+\sin^2 r}{2}.
\end{aligned}
$$

By definition, we know the three kinds of decorrelating capabilities of Unruh channel are

$$
\begin{aligned}
D^I(r) :=&D(\mathcal{E})=I(\rho^{AR})-I(\rho^{A,I}),\\
D_c^I(r) :=&D_c(\mathcal{E})=C(\rho^{AR})-C(\rho^{A,I}),\\
D_q^I(r) :=&D_q(\mathcal{E})=Q(\rho^{AR})-Q(\rho^{A,I}),
\end{aligned}
$$

and the decorrelating capabilities of the complementary Unruh channel are

$$
\begin{aligned}
D^{II}(r) :=&D(\mathcal{E}_c)=I(\rho^{AR})-I(\rho^{A,II}),\\
D_c^{II}(r) :=&D_c(\mathcal{E}_c)=C(\rho^{AR})-C(\rho^{A,II}),\\
D_q^{II}(r) :=&D_q(\mathcal{E}_c)=Q(\rho^{AR})-Q(\rho^{A,II}).
\end{aligned}
$$

We plot the decorrelating capabilities of Unruh channel and its complementary channel in Fig. 3 and Fig. 4, respectively.

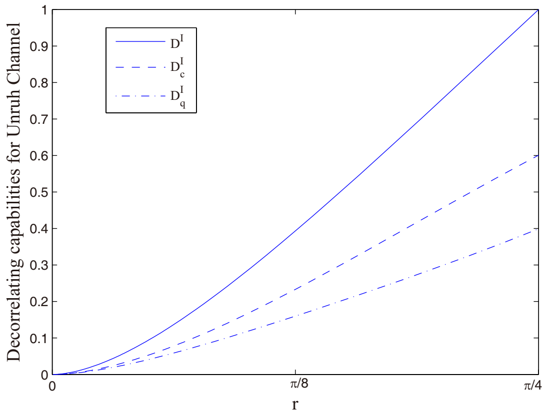


Figure 3. The decorrelating capabilities for Unruh channel.

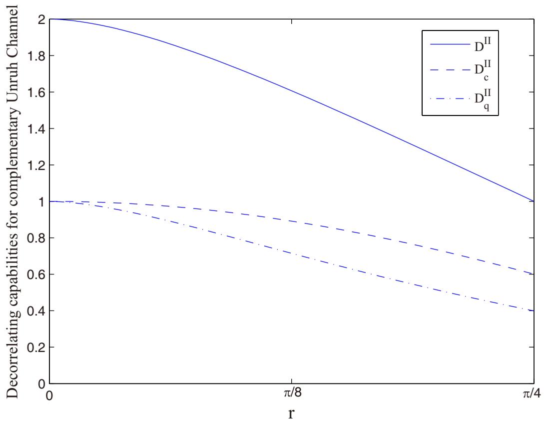


Figure 4. The decorrelating capabilities for complementary Unruh channel.

## 4 Measurement-Induced Correlations of Unruh Effect

First recall the definition of measurement-induced correlations for the measurement $\mathcal{E}$ performed on the state $\rho = \rho^B$ in system B[18]. Let $(U, |C\rangle)$ be an unitary realizations of $\mathcal{E}$, that is,

$$\mathcal{E}(\rho) = \mathrm{tr}_C U(\rho \otimes |C\rangle\langle C|)U^\dagger,$$

and

$$\sigma^{BC} := U(\rho \otimes |C\rangle\langle C|)U^\dagger.$$

By virtue of the classical correlations and quantum correlations of the final system-apparatus state we define measurement-induced total, classical, and quantum correlations as

$$I_\rho(\mathcal{E}) := I(\sigma^{BC}),\ C_\rho(\mathcal{E}) := C(\sigma^{BC}),\ Q_\rho(\mathcal{E}) := Q(\sigma^{BC}),$$

respectively. When $\rho = \mathbf{1}/d^B$ is the maximally mixed state, we denote the corresponding measurement-induced correlations as $I(\mathcal{E})$, $C(\mathcal{E})$ and $Q(\mathcal{E})$, respectively.

When $c_1 = 1$, $c_2 = -1$, $c_3 = 1$, the initial state is maximally entangled state. $I(\rho^{\mathrm{I,II}})$ is just the measurement-induced total correlations by Unruh Channel. Hence, the measurement-induced total, classical, and quantum correlations can be calculated as

$$\begin{aligned}
I(\mathcal{E}) =& I(\rho^{I,II})\\
=& S(\rho^{I}) + S(\rho^{II}) - S(\rho^{I,II})\\
=& H(\{\lambda^I, 1-\lambda^I\}) + H(\{\lambda^{II}, 1-\lambda^{II}\}) - H(\{1/2, 1/2, 0, 0\})\\
=& -\frac{\cos^2 r}{2}\log_2\frac{\cos^2 r}{2} - \frac{1+\sin^2 r}{2}\log_2\frac{1+\sin^2 r}{2}\\
& -\frac{\sin^2 r}{2}\log_2\frac{\sin^2 r}{2} - \frac{1+\cos^2 r}{2}\log_2\frac{1+\cos^2 r}{2} - 1,\\
C(\mathcal{E}) =& C(\rho^{I,II})\\
=& S(\rho^{I}) - E(\rho^{A,I})\\
=& -\frac{\cos^2 r}{2}\log_2\frac{\cos^2 r}{2} - \frac{1+\sin^2 r}{2}\log_2\frac{1+\sin^2 r}{2}\\
& +\frac{1+\sin r}{2}\log_2\frac{1+\sin r}{2} + \frac{1-\sin r}{2}\log_2\frac{1-\sin r}{2},\\
Q(\mathcal{E}) =& Q(\rho^{I,II})\\
=& I(\rho^{I,II}) - C(\rho^{I,II})\\
=& S(\rho^{II}) - S(\rho^{I,II}) + E(\rho^{A,I})\\
=& -\frac{\sin^2 r}{2}\log_2\frac{\sin^2 r}{2} - \frac{1+\cos^2 r}{2}\log_2\frac{1+\cos^2 r}{2} - 1\\
& -\frac{1+\sin r}{2}\log_2\frac{1+\sin r}{2} - \frac{1-\sin r}{2}\log_2\frac{1-\sin r}{2},
\end{aligned}$$

respectively.

We plot the measurement-induced correlations for Unruh effect in Fig. 5.

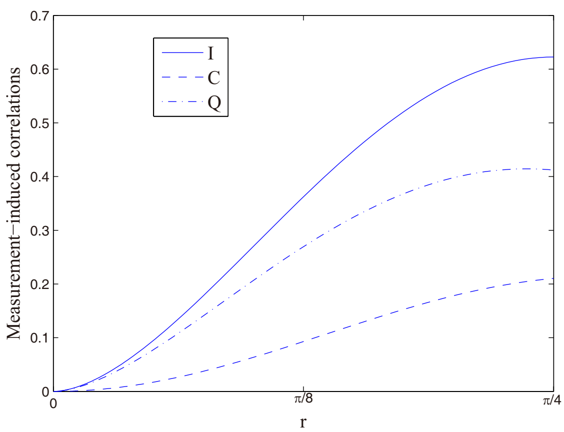


Figure 5. Measurement-induced correlations for Unruh effect.

## 5 Conclusions

Recently, the issue of how the Unruh effect affects the information content (or more specifically, correlation measures) in quantum states becomes one of the central topics in relativistic quantum information thoery. However, most of the results are restricted to the case that the initial states between the two modes shared by the inertial observer and the accelerated observer are maximally entanglement states. In this work, we investigate the more general initial states, any Bell-diagonal states. We first introduce the information loss induced by Unruh effect when the initial states are any Bell-diagonal states, and give their analytic expressions. On the other hand, we use the concepts of the decorrelating capabilities and measurement-induced correlations of quantum channels to reinterpret the changes of correlations induced by Unruh effect in a free Dirac field, which may shed new light on the understanding of Unruh effect from the information theory viewpoint.

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